

# Possible approach to improve sensitivity of a Michelson interferometer

Jian Fu

*State Key Lab of Modern Optical Instrumentation,*

*Department of Optical Engineering,*

*Zhejiang University, Hangzhou 310027, China*

## Abstract

We propose a possible approach to achieve an  $1/N$  sensitivity of Michelson interferometer by using a properly designed random phase modulation. Different from other approaches, the sensitivity improvement does not depend on increasing optical powers or utilizing the quantum properties of light. Moreover the requirements for optical losses and the quantum efficiencies of photodetection systems might be lower than the quantum approaches and the sensitivity improvement is frequency independent in all detection band.

PACS numbers: 04.80.Nn, 42.87.Bg, 07.60.Ly

Michelson interferometer plays an important role in ultimate sensitivity measurements. Especially, large-scale interferometric gravitational wave detectors [1] such as the Laser Interferometric Gravity-wave Observatory (LIGO) is a Michelson interferometer with arm cavities and power recycling [2]. In the measurements, the ultimate sensitivity is conventionally bounded by the quantum nature of the electromagnetic field. It has been shown that the so-called shot noise is due to the vacuum fluctuations coupled to the interferometer and the radiation pressure noise is due to the random motion of the mirrors induced by the radiation pressure fluctuations [3]. However, these standard quantum limits (SQL) are not as fundamental as the Heisenberg limits, and can be beaten by using quantum entanglement [4] and squeezing [5]. Benefiting from a nonlocal correlation, quantum entanglement can achieve an  $1/N$  sensitivity with a  $\sqrt{N}$  precision improvement ( $N$  being the number of photons employed) over the classical strategies [4]. But there is an enormous difficulty in the quantum-enhanced measurement, that is usually very complicated to realize multi-particle quantum entanglement even as few as 5 or 6 particles [6]. Other quantum approaches using squeezed states [5, 7] or Fock states [8] can also achieve the  $1/N$  sensitivity [4]. However, these approaches for sensitivity enhancement beyond the SQL require optical losses to be very low and the quantum efficiencies of photodetection systems to be very high [9].

Recently, “mode-entangled states” based on the transverse modes of classical optical fields propagating in multimode waveguides are proposed as classical simulations of quantum entangled states [10]. It is interesting that the mode-entangled states can also exhibit the nonlocal correlation, such as the violation of Bell’s inequality. By using the classical simulation of  $N$ -particle quantum entangled states, an interferometer that can beat the standard quantum limit was proposed in Ref. [11]. Similar to the quantum-enhanced measurements, the interferometer can also achieve an  $1/N$  sensitivity with a  $\sqrt{N}$  precision improvement over ordinary interferometers. It is noticeable that the nonlocal correlation similar to quantum entanglement might be caused by a random phase mechanism [12]. It might also be a possible mechanism for improving the sensitivity and precision of interferometers [11].

In this paper, we propose a possible approach to improve the sensitivity of Michelson interferometer by using a random phase modulation. In the scheme, we will divide each sampling period into  $N$  time slots. The measurement in each time slot can be regarded as an independent measurement. By modulating properly designed random phase sequences in time slots, the  $N$  measurements will become similar to the measurements employed  $N$ -

field mode-entangled states [11]. At last, by performing a correlation analysis on these measurement results, it is in principle possible to achieve an  $1/N$  sensitivity.

Before turning to a detailed analysis of the scheme, it is useful to discuss an ordinary Michelson interferometer. A classical coherent beam with  $N_p$  average photons enters impinges on a semi-transparent mirror (i.e. a beam splitter), which divides it into a reflected and a transmitted beam. These two beams travel along different paths and then are reflected back by two mirrors. At last they are recombined by the same beam splitter. By measuring the intensity difference of the two output beams, one can recover the phase  $\theta = 4\pi\Delta L/\lambda$  ( $\Delta L$  being the path difference) between the two optical paths with a statistical error proportional to  $1/\sqrt{N_p}$ . This is the SQL due to the quantized nature of electromagnetic field and the Poissonian statistics of classical light.

Here, a Michelson interferometer with a minor change to achieve an  $1/N$  sensitivity is proposed, and the scheme is shown in Fig. 1. In the scheme, a phase modulator  $\varphi$  is mounted on one of the optical paths. This is only a different between the scheme and an ordinary Michelson interferometer. Suppose that a time dependent extremely small phase  $\theta = 4\pi\Delta L/\lambda$  has an upper frequency limit  $f_u$ . According to Nyquist sampling theory, we can determine a sampling period  $T_s = 1/2f_u$  to measure the phase difference. Then we divide each sampling period into  $N$  time slots. The measurement in each time slot can be regarded as an independent measurement. Further, we divide each time slot into  $M$  phase units. In each phase unit, a random phase  $\varphi$  uniformly distributed in  $[0, 2\pi]$  is modulated by the phase modulator. Thus, we obtain  $N$  random phase sequences of length  $M$  that can be written as  $\{\varphi_j^i, (i = 1 \dots N, j = 1 \dots M)\}$ . By measuring the intensity difference  $I$  of the two output beams in each phase unit, we can obtain

$$I_j^i = \frac{N_p}{MN} \cos(\theta + \varphi_j^i). \quad (1)$$

Due to the random phase  $\varphi_j^i$  uniformly distributed in  $[0, 2\pi]$ , the intensity difference  $I_j^i$  is also randomly varied in the range of  $\pm N_p/MN$ . Thus, we can obtain  $N$  random intensity difference sequences  $\{I_j^i, (i = 1 \dots N, j = 1 \dots M)\}$ . The relation of the time slots, the phase and intensity difference sequences is shown in Fig. 2.

In order to extract the information of  $\theta$  from the random intensity difference sequences

$\{I_j^i\}$ , we properly design the random phase sequence of the  $N$ th time slot as

$$\varphi_j^N = 2\pi - \left( \sum_{i=1}^{N-1} \varphi_j^i \right) \bmod 2\pi. \quad (2)$$

Then a correlation analysis is performed on the intensity difference sequences, and the correlation function can be written as

$$\begin{aligned} S_N(\theta) &= \frac{1}{M} \sum_{j=1}^M \left[ \prod_{i=1}^N I_j^i \right] \\ &= \frac{1}{M} \left( \frac{N_p}{MN} \right)^N \sum_{j=1}^M \left[ \prod_{i=1}^N \cos(\theta + \varphi_j^i) \right] \\ &= \frac{2}{M} \left( \frac{N_p}{2MN} \right)^N \sum_{j=1}^M \left\{ \cos \left( N\theta + \sum_{i=1}^N \varphi_j^i \right) + \cos \left[ (N-2)\theta + \sum_{i=1}^N \varphi_j^i - 2\varphi_j^1 \right] + \dots \right\}. \end{aligned} \quad (3)$$

By using  $\sum_{i=1}^N \varphi_j^i = 2\pi$  obtained from Eq. (2), we can obtain

$$S_N(\theta) = 2 \left( \frac{N_p}{2MN} \right)^N \left\{ \cos(N\theta) + \frac{1}{M} \sum_{j=1}^M [\cos(N\theta - 2\theta - 2\varphi_j^1) + \dots] \right\}, \quad (4)$$

where the expression of  $\sum_{j=1}^M \dots$  contains  $2^{N-1} - 1$  terms that are all random-phase cosine functions. If the random phases are uniformly distributed in  $[0, 2\pi]$ , the sum of the random-phase cosine terms equals zero.

By using orthogonal pseudo-random number (PN) sequence techniques, we can completely eliminate the terms of  $\sum_{j=1}^M \dots$ . In all PN sequences, maximal-length linear feedback shift-register sequence (M-sequence) is often used as spread-spectrum sequence and can simultaneously exhibit good weight distribution and correlation properties. Here, we use the M-sequence technique to generate the random phase sequences. After given a generating polynomial of order  $n$ ,  $2^n - 1$  different M-sequences of length  $2^n - 1$  can be obtained by shifting bits. By properly chosen from these sequences, the  $N - 1$  random sequences of length  $M = 2^n$  (a code 0 or 1 added to balance their numbers) can be obtained. Then, by alternately mapping the codes 0 to the phases 0 and  $\pi$ , and 1 to  $\pi/2$  and  $3\pi/2$ , we obtain the  $N - 1$  random phase sequences in which the phases are assigned with four discrete values 0,  $\pi/2$ ,  $\pi$  and  $2\pi/3$ . By using Eq. (2), the random phase sequence of the  $N$ th time slot can be obtained. By using the  $N$  random phase sequences, we eliminate the terms of  $\sum_{j=1}^M \dots$  and obtain the correlation function

$$S_N(\theta) = 2 \left( \frac{N_p}{2MN} \right)^N \cos(N\theta). \quad (5)$$

This result shows a sensitivity of the order  $1/N$  for the measurements of small phase  $\theta$ .

In the quantum-enhanced measurement, a  $\sqrt{N}$  precision improvement over the classical strategies can be achieved, with the concomitant improvement in sensitivity. However, in the scheme, the  $\sqrt{N}$  precision improvement might not be achieved due to the complication of the random phase modulation. Its precision is mainly limited by two unavoidable sources of errors. The first source is the shot noise. In each phase unit, the phase error is proportional to  $\sqrt{MN/N_p}$  due to  $N_p/MN$  average photons. In the correlation analysis, the variance associated with  $\frac{1}{M} \sum_{j=1}^M \dots$  is given by  $\sqrt{N/N_p}$ . By using the correlation function, the phase error  $\Delta\theta$  can be obtained from error propagation,  $\Delta\theta = \Delta S_N / |\partial S_N / \partial \theta|$ , it is easy to see that it scales as  $1/N$ . Therefore we can obtain the shot noise limit as  $\Delta\theta = 1/\sqrt{NN_p}$ . The second source is a phase error induced by the phase modulation. When a light beam is modulated by an electro-optical phase modulator such as LiNbO<sub>3</sub> modulator, the phase error  $\Delta\varphi$  is induced by the voltage fluctuations of control signals. The phase error will lead to the intensity fluctuations due to the terms of  $\sum_{j=1}^M \dots$  and the phase sum error  $\sum_{i=1}^N \varphi_j^i = 2\pi \pm N\Delta\varphi$ . The phase precision limit can be estimated to be  $(\sqrt{2^{N-1}(N-1)/MN} + 1)\Delta\varphi$ . At last, we can obtain the overall phase error

$$\Delta\theta = \frac{1}{\sqrt{NN_p}} + \left( \sqrt{\frac{2^{N-1}(N-1)}{MN}} + 1 \right) \Delta\varphi. \quad (6)$$

Although, a  $\sqrt{N}$  precision improvement of the shot noise limit is achieved in the scheme, a new phase error is induced by the phase modulation.

In order to reduce the new phase error  $\Delta\varphi$ , we propose a configuration of optical phase-locked loop (OPLL) as shown in Fig. 3. The OPLL technique is generally applied to control the phase error of two different lasers in optical PSK homodyne transmission systems and easily realized without any rigorous requirements for optical losses and photodetection systems [13, 14, 15]. In this scheme, two optical beams are split from the fore-and-aft places of the phase modulator and combined by a semi-transparent mirror. After measured the intensity difference, the signal is further processed by a gain adjustable loop amplifier and a loop filter. To complete the loop, the loop filter is connected to a control voltage generator that generates suitable voltages to control the phase modulator. The performance of the OPLL depends on the properties of the input optical beams and loop design. In this scheme, the shot noise might be the main source of the phase error. When the loop is locked, the phase error contributed by the shot noise can be written as  $\Delta\varphi = \sqrt{N_0 B_L}/A$ , where  $N_0$

is the power spectral density of the shot noise,  $B_L$  is the one sided loop bandwidth, and  $A = 2rRP$  is the gain of the intensity difference detectors with the receiver transimpedance  $r$ , the photodetector responsivity  $R$  and the received optical powers  $P$ . The phase locking action can dramatically suppress the phase error for a narrow loop bandwidth and a high value of  $A^2/N_0$  [16]. As reported in Ref. [13], the phase error of two different lasers might be controlled below  $0.3^\circ$  for signal powers  $P_s \geq -62\text{dBm}$ . In this scheme, the phase error limit should be less than that for without the influence of frequency perturbation. Further refinement of the OPLL, such as optimal loop bandwidth, might allow to improve the phase precision until nearly reaching the  $\sqrt{N}$  precision improvement over ordinary interferometers.

In many measurements with optical interferometers, the ultimate sensitivity is required. By using this scheme, we can achieve an  $1/N$  sensitivity with few changes on the optical interferometers. Consider the detection band of  $40\text{Hz}$  to  $5\text{kHz}$ , the sampling period  $T_s = 0.1\text{ms}$  can be determined. If an  $1/10$  sensitivity is required, each sampling period is divided into 10 time slots. After given the 8th-order generating polynomial  $f(x) = 1 + x^2 + x^3 + x^4 + x^8$ , 255 M-sequences of length 255 are obtained. By properly chosen and calculated, 10 random phase sequences of length  $M = 256$  are obtained to eliminate the terms of  $\sum_{j=1}^M \dots$ . Then, we can obtain that the rate requirement for the phase modulator is  $25.6\text{MHz}$ . It is very easy to reach the requirement for  $\text{LiNbO}_3$  modulator that is a fairly mature commercial technology. Further, we can estimate the phase error  $\Delta\theta \sim 0.7^\circ$  induced by the phase modulation (assumed the phase error as Ref. [13]). After careful study, we find the generating polynomial order  $n \geq N - 2$ . This leads to a difficulty to further improve the sensitivity, that is the number of phase units require exponentially increasing with the sensitivity improvement. It might be reduced by using some new PN sequences.

In this paper, we have discussed a possible approach to improve the sensitivity for a Michelson interferometer. Different from other approaches, the sensitivity improvement does not depend on increasing optical powers or utilizing the quantum properties of light. Moreover its requirements for the optical losses and the quantum efficiencies of photodetection systems might be lower than the quantum approaches and the sensitivity improvement is frequency independent in all detection band. The approach might be applicable to some laser interferometers for gravity-wave detection. Most gravity-wave laser interferometers, such as LIGO, rely on sophisticated power and signal recycling schemes with carrier/sideband measurement signatures. Therefore, a readout and control scheme compatible with both

the correlation analysis and the recycle of power and signal deserves further study. Besides this application, the approach can be generally applied to various types of interferometers without any rigorous requirements of optical powers or the quantum properties of light.

This work was supported by the National Natural Science Foundation of China under Grant No. 60407003.

- 
- [1] K. Thorne, *Rev. Mod. Phys.* **52**, 285 (1980); L. Ju, D. G. Blair, and C. Zhao, *Rep. Prog. Phys.* **63**, 1317 (2000).
  - [2] B. Barish and R. Weiss, *Phys. Today* **52**, 44 (1999); A. Abramovici, and et al., *Science* **256**, 325 (1992).
  - [3] C. M. Caves, *Phys. Rev. Lett.* **45**, 75 (1980); V. B. Braginsky, and Y. I. Vorontsov, *Sov. Phys. Usp.* **17**, 644 (1975); W. A. Edelstein, J. Hough, J. R. Pugh, J. Martin, *J. Phys. E* **11**, 710 (1978).
  - [4] V. Giovannetti, S. Lloyd, L. Maccone, *Science* **306**, 1330 (2004).
  - [5] M. Xiao, Ling-An Wu, and H. J. Kimble, *Phys. Rev. Lett.* **59**, 278 (1987).
  - [6] Z. Zhao, and et al., *Nature* **430**, 55 (2004).
  - [7] R. S. Bondurant and J. H. Shapiro, *Phys. Rev. D* **30**, 2548 (1984).
  - [8] M. J. Holland and K. Burnett, *Phys. Rev. Lett.* **71**, 1355 (1993); J. Jacobson, and et al., *Phys. Rev. Lett.* **74**, 4835 (1995).
  - [9] A. Buonanno and Y. Chen, *Phys. Rev. D* **69**, 102004 (2004); A. Buonanno and Y. Chen, *Phys. Rev. D* **64** 042006 (2001); T. Kim, Y. Ha, and et al., *Phys. Rev. A* **60**, 708 (1999).
  - [10] J. Fu, Z. J. Si, S. F. Tang, and et al., *Phys. Rev. A* **70**, 042313 (2004).
  - [11] J. Fu, quant-ph/0604103.
  - [12] J. Fu and X. Zhang, quant-ph/0506125.
  - [13] L. G. Kazovsky, *J. Lightwave Technol.* **8**, 1414 (1990); D. A. Atlas and L. G. Kazovsky, *IEEE Photon. Technol. Lett.* **2**, 367 (1990).
  - [14] S. Norimatsu, K. Iwashita, and K. Noguchi, *Electron. Lett.* **26**, 648 (1990); S. Norimatsu and K. Iwashita, *J. Lightwave Technol.* **9**, 1367 (1991); B. Wandernoth, *Electron. Lett.* **28**, 387 (1992); J. M. Kahn, *IEEE Photon. Technol. Lett.* **1**, 340 (1989).

- [15] D. J. Malyon, Electron. Lett. **20**, 281 (1984); L. G. Kazovsky, J. Lightwave Technol. **LT-4**, 182 (1986); D. J. Malyon, D. W. Smith, and R. Wyatt, Electron. Lett. **22**, 421 (1986); J. M. Kahn, and et al., IEEE Photon. Technol. Lett. **2**, 285 (1990).
- [16] M. Z. Win, C-C. Chen, and R. A. Scholtz, Proceedings of SPIE **1417**, 40 (1991).



**Fig. 1:** The scheme of the Michelson interferometer to achieve higher sensitivity.

**Fig. 2:** The relation of the time slots, the phase and intensity difference sequences.

**Fig. 3:** The configuration of optical phase-locked loop to control the phase error, LF: Loop Filter, CVG: Control Voltage Generator, PNG: Pseudo-random Number Generator, PM: Phase Modulator.